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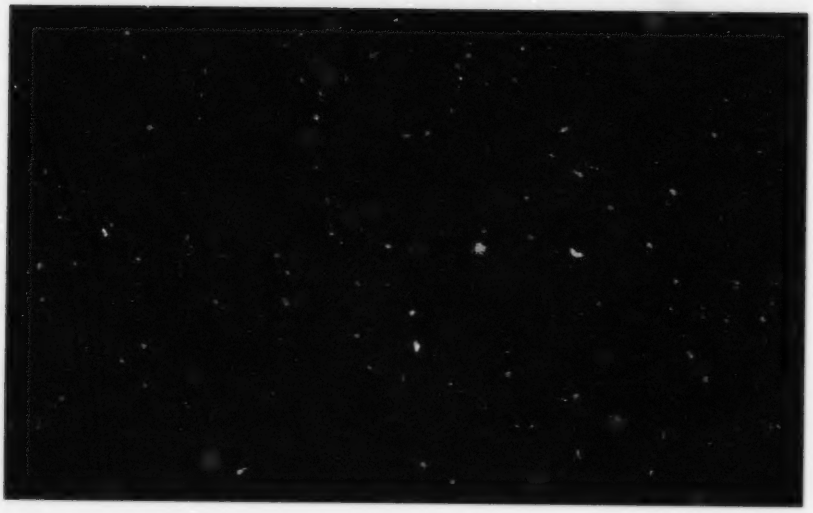

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Adverse Selection in Insurance Markets: A Selective Survey

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ABSTRACT

In this survey we present some of the more significant results in the literature on adverse selection in insurance markets. Section 1 discusses the monopoly model introduced by Stiglitz (1977) for the case of single-period contracts and extended by many authors to the multi-period case. The introduction of multi-period contracts raises many issues that are discussed in detail : time horizon, discounting, commitment of the parties, contract renegotiation and accidents underreporting. Section 2 covers the literature on competitive contracts. The analysis becomes more complicated since insurance companies must take into account competitive pressures when they set incentives contracts. As pointed out by Rothschild and Stiglitz (1976), there is not necessarily a Cournot-Nash equilibrium in presence of adverse selection. However, market equilibrium can be sustained when principals anticipate competitive reactions to their behaviour or when they adopt strategies that differ from the pure Nash strategy. Multi-period contracting is discussed. We show that different predictions on the evolution of insurer profits over time can be obtained from different assumptions concerning the sharing of information between insurers about individual's choice of contracts and accidents experience. The role of commitment and renegotiation between the parties to the contract are important. We then discuss how risk categorization can be use to improve resource allocation under adverse selection. Finally, the last section introduces models that consider moral hazard and adverse selection simultaneously. A short conclusion summarizes the main results in recent literature and discusses some avenues of future research.

Key words : adverse selection, insurance markets, monopoly, competitive contracts, self-selection mechanisms, single-period contracts, multi-period contracts, commitment, contract renegotiation, accidents underreporting, risk categorization.

RÉSUMÉ

Dans cette revue, nous présentons quelques résultats importants de la littérature sur les contrats d'assurance en présence de sélection adverse. La section 1 discute le modèle du monopole introduit par Stiglitz (1977) pour les contrats à une période et étendu par plusieurs auteurs aux contrats à plusieurs périodes. L'introduction des contrats à plusieurs périodes soulève plusieurs considérations qui sont discutées en détail : l'horizon temporel, l'actualisation, les engagements des parties au contrat, la renégociation, les déclarations d'accidents. La section 2 couvre la littérature sur les contrats concurrentiels. L'analyse devient vite plus compliquée car les assureurs doivent tenir compte des pressions concurrentielles lorsqu'ils écrivent les contrats. Le résultat classique de Rothschild et Stiglitz (1976) montre qu'il n'y a pas nécessairement un équilibre Cournot-Nash en présence de sélection adverse. Par contre, un équilibre peut être obtenu lorsque les assureurs anticipent les réactions des concurrents à leur comportement. Les contrats à plusieurs périodes sont aussi discutés. Nous montrons que différentes prédictions des profits des assureurs dans le temps peuvent être obtenues si des hypothèses différentes concernant le partage de l'information des assureurs sur l'expérience des assurés sont considérées. Le rôle des engagements des parties au contrat et de la renégociation sont aussi importants. Nous montrons également comment la classification des risques peut améliorer l'allocation des ressources en présence de sélection adverse. Finalement, la dernière section du texte introduit des modèles qui traitent des problèmes de risque moral et de sélection adverse de façon simultanée. Une courte conclusion résume les principaux résultats et discute de quelques avenues de recherche future.

Mots clés : sélection adverse, marchés d'assurance, monopole, contrats concurrentiels, mécanismes d'auto-sélection, contrats à une période, contrats à plusieurs périodes, engagements des parties au contrat, renégociation, déclaration d'accidents, classification des risques.

INTRODUCTION

Without asymmetric information and under the standard assumptions of insurance models that we shall use in this article (same attitude toward risk and same risk aversion for all individuals in all classes of risk, one source of risk, risk neutrality on the supply side, no transaction cost in the supply of insurance, and no moral hazard), a Pareto optimal solution is characterized by full insurance coverage for all individuals in each class of risk. Each insured sets his optimal consumption level according to his certain wealth. No other financial institution is required to obtain this level of welfare and both risk categorization and self-selection mechanisms are redundant. Moreover, there is no need for multi-period insurance contracts since they are not superior to a sequence of one-period contracts. Finally, the two standard theorems of welfare economics hold and market prices of insurance are equal to the corresponding social opportunity costs.

In insurance markets, adverse selection results from asymmetric information between the insured (agent) and the insurer (principal). The insureds are heterogeneous with respect to their expected loss and have more information than the insurance company which is unable to differentiate between risk types. Naturally, the high risk individual has no incentive to reveal his true risk which is costly to observe by the insurer. As pointed out by Arrow, a pooling of risks is often observed in insurance markets. "In fact, however, there is a tendency to equalize rather than to differentiate premiums... This constitutes, in effect, a redistribution of income from those with a low propensity of illness to those with a high propensity..." (Arrow, 1963; p. 964).

Akerlof (1970) showed that if all insurers have imperfect information on individual risks, an insurance market may not exist, or if it exists, it may not be efficient. He proposed an explanation of why, for example, people over 65 have great difficulty in buying medical insurance : "the result is that the average medical condition of insurance applicants deteriorates as the price level rises - with the result that no insurance sales may take place at any price" (1970; p. 492). The seminal contributions of Akerlof and Arrow have generated a proliferation of models on adverse selection. In this survey we shall, however, confine attention to a limited subset. Many authors have proposed mechanisms to reduce the inefficiency associated with adverse selection : the "self-selection mechanism" in one period

contracts which induces policyholders to reveal hidden information by selection from a menu of contracts, (Rothschild and Stiglitz, 1976; Stiglitz, 1977; Wilson, 1977; Miyazaki, 1977; Spence, 1978; Hellwig, 1986), the "categorization of risks" (Hoy, 1982; Crocker and Snow, 1985, 1986), and multi-period contracts (Dionne, 1983, Dionne and Lasserre, 1985, 1987; Kunreuther and Pauly, 1985; Cooper and Hayes, 1987; Hosios and Peters, 1989; Nilssen, 1990; Dionne and Doherty (1991)). All of them address private market mechanisms. In the first case, insurers offer a menu of policies with different prices and quantity levels so that different risk types choose different insurance policies. Pareto improvements for resource allocation with respect to the single contract solution with an average premium to all clients can be obtained. In the second case, insurers use imperfect information to categorize risks and, under certain conditions, it is also possible to obtain Pareto improvements for resource allocation. In the third case, insurers use the information related to the past experience of the insured as a sorting device (i.e. to motivate high risk individuals to reveal their true risk ex ante).

In this survey we present some of the more significant results in the literature on adverse selection in insurance markets. Section 1 discusses the monopoly model introduced by Stiglitz (1977) for the case of single-period contracts and extended by many authors to the multi-period case. The introduction of multi-period contracts raises many issues that are discussed in detail : time horizon, discounting, commitment of the parties, contract renegotiation and accident underreporting. Section 2 covers the literature on competitive contracts. The analysis becomes more complicated since insurance companies must take into account competitive pressures when they set incentives contracts. As pointed out by Rothschild and Stiglitz (1976) there is not necessarily a Cournot-Nash equilibrium in presence of adverse selection. However, market equilibrium can be sustained when principals anticipate competitive reactions to their behaviour or when they adopt strategies that differ from the pure Nash strategy. Multi-period contracting is discussed. We show that different predictions on the evolution of insurer profits over time can be obtained from different assumptions concerning the sharing of information between insurers about individual's choice of contracts and accidents experiences. The role of commitment and renegotiation between the parties to the contract are important. We then discuss how risk categorization can be used to improve resource allocation under adverse selection. As shown recently by Bond and Crocker (1990), risk categorization has two potential effects : 1) it may improve efficiency

and 2) it may mitigate nonexistence of equilibrium problems. Finally, the last section introduces models that consider moral hazard and adverse selection simultaneously. A short conclusion summarizes the main results and discusses some avenues of future research.

Before proceeding let us comment briefly on some standard assumptions. We assume that all individuals maximize expected utility. The utility functions of the individuals in each risk group are identical, strictly concave and satisfy the von Neumann-Morgenstern axioms. Utility is time independent, time additive and state-independent. In many models there is no discounting. Individuals start each period with a given wealth, W, which is non random. To avoid problems of bankruptcy, the value of the risky asset is lower than W. All risk in the individual's portfolio is assumed to be insurable. Income received in a given period is consumed in that period; effectively there is no saving and no banking. Insurers are risk neutral and maximize the value of their cash flows or profits. Insurers write exclusive insurance contracts and there are no transaction costs in the supply of insurance. Finally, the insureds are assumed to be unable to influence either the probabilities of accident or the damages due to accidents; this rules out any problem of moral hazard.

As indicated above, the design of optimal contracts when moral hazard and adverse selection are present simultaneously is discussed in the last section of the paper. Recently, the two information problems have been integrated by some authors into a single model (Laffont and Tirole, 1986; Guesnerie, Picard, Rey, 1988) where all the parties to the contract are risk neutral. However, such contributions cannot be applied directly to insurance contracts where at least one party to the contract is risk averse.

To simplify the presentation we explicitly assume that insurers are risk neutral. An equivalent assumption is that insurers are well diversified in the sense that much of their total risk is diversified by their own equity holders in the management of their personal portfolios. The presence of transaction costs would not affect the qualitative conclusions concerning the effects of adverse selection on resource allocation in insurance markets. However, proportional transaction costs (or proportional loadings) are sufficient to explain partial insurance coverage and their explicit introduction in the analysis would modify some conclusions in the reference models. For example, each individual in each class of risk

would buy less than full insurance in presence of full information and the introduction of adverse selection will decrease further the optimal coverage for the low risk individuals.

The presence of many sources of non insurable risks or of many risky assets in individual portfolios is also an empirical fact that is not considered in the models. As long as these risks are independent, the conclusions should not be affected significantly. However, the optimal portfolio and insurance decisions in the presence of many correlated risks and asymmetrical information in one or in many markets is still an open question in the literature.

In reality, we observe that banks coexist with insurers who offer multi-period insurance contracts. The presence of saving and banking may change the conclusions obtained for multi-period contracts under asymmetrical information. Particularly, it may modify accidents reporting strategies and commitment to the contracts. However, with few exceptions (Allen (1985), moral hazard; Dionne and Lasserre (1987), adverse selection; Fudenberg, Holmstrom and Migrom (1986), moral hazard) research on principal-agent relationships has not envisaged the simultaneous presence of several alternative types of institutions.

The assumption of exclusive insurance contracting in discussed in Section 2 and some aspects of the discounting issues are discussed in Section 1. There remains the assumptions on the utility function. Although the theory of decision making under uncertainty has be challenged since its formal introduction by von Neumann and Morgenstern (Machina, 1987), it has produced very useful analytical tools for the study of optimal contracts such as, for example, optimal insurance coverage and the associated comparative statics, as well as the design of optimal contracts under moral hazard or the characterization of optimal insurance policies under adverse selection. In fact, very few contributions use non-linear models in insurance literature (see however Karni, 1990) and none of these has addressed the adverse selection. In this survey we then limit the discussion to the linear expected utility model. We also assume that utility functions are not function of the states of the world and that all individuals in all classes of risks have the same level of risk aversion. These assumptions are not necessary to get the desired results but permit the discussion to focus on differences in the risk types.

1. MONOPOLY

1.1 Public Information

There are two possible states of the world ($x \in \{n,a\}$) : state (n), "no accident" having the probability $(1 - p_i)$ and state (a), "accident" having the probability $0 < p_i < 1$. Consumers differ only by their probability of accident. For simplicity, there are two types of risk in the economy ($i \in \{H,L\}$ for high and low risk) with $p_H > p_L$. Each consumer owns a risky asset with monetary value D(x); D(a) = 0 in state (a) and D(n) = D in state (n). Therefore the expected damage for a consumer of type i (E_i D(x)) is p_i D.

Under public information and without transaction cost, a risk neutral private monopoly[1] would offer insurance coverage (net of premium) (β_i) for an insurance premium (α_i) such that a consumer will be indifferent between purchasing the policy and having no insurance (Stiglitz, 1977). In other words, the private monopolist maximizes his total profit over α_i, β_i and λ_i :

Problem 1 : $$\max_{\alpha_i, \beta_i, \lambda_i} \quad \Sigma\, q_i((1 - p_i)\, \alpha_i - p_i\beta_i) \qquad (1)$$

under the individual rationality (or participating) constraints

$$V(C_i|p_i) - V(C^0|p_i) \geq 0 \qquad i = H,L \qquad (2)$$

where :

$V(C_i|p_i)$ is the expected utility under the contract $C_i = \{\alpha_i,\beta_i\}$:

$$V(C_i|p_i) = p_i\, U(W - D + \beta_i) + (1 - p_i)\, U(W - \alpha_i);$$

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[1]For an analysis of several reasons why a monopoly behaviour in insurance markets should be considered, see Dahlby (1987). For examples of markets with a monopoly insurer see D'Arcy and Doherty (1990) and Dionne and Vanasse (1988). The result of this section also applies to a cartelized market.

$U(\cdot)$ is a twice differentiable, strictly increasing and strictly concave function of final wealth ($U'(\cdot) > 0$, $U''(\cdot) < 0$);

W is non random initial wealth;

C^0 denotes self-insurance; $C^0 = \{0,0\}$ implies that $V(C^0|p_i) \equiv p_i\, U(W - D) + (1 - p_i)\, U(W)$; $V(C^0|p_i)$ is the reservation utility. Below this level, individuals will self insure.

q_i is the number of policies sold to consumers of type i;

λ_i is a Lagrangian multiplier for constraint (2).

It is well known that full insurance, $\beta_i^* = D - \alpha_i^*$ (for i = L,H), is the solution to the above problem and that (2) is binding for both classes of risk which means that

$$V(C_i^*|p_i) = V(C^0|p_i) \qquad i = H,L$$

or $\alpha_i^* = p_i D + z_i^*$,

where z_i^* is the maximum unit-profit (or the maximum risk premium) on each policy. In other words z_i^* solves : $U(W - p_i D - z_i^*) = p_i U(W - D) + (1 - p_i)\, U(W)$.

The private monopoly extracts all the consumer surplus. However, there is no efficiency cost since each individual buys full insurance as under perfect competition[2]. This is the classical result that Pareto efficient risk sharing between a risk averse agent and a risk-neutral principal shifts all the risk to the principal. To sum up we can write :

Proposition 1 : In presence of public information about insureds' underlying risk, an optimal contract between a private monopolist and any individual of type i is characterized by :

a) full insurance coverage, $\beta_i^* = D - \alpha_i^*$;

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[2]As in the perfect discrimination case, the monopolist charges a price of insurance to each consumer equal to marginal cost. All potential consumer surplus are connected into monopoly profits so there is no dead weight loss.

b) no consumer surplus, $V(C_i^*|p_i) = V(C^0|p_i)$.

Both solutions are shown at C_H^* and C_L^* in Figure 1 where C^0 is the "initial endowment" or self-insurance situation and where the vertical axis is wealth in the accident or loss state.

(Insert Figure 1 here)

Any point to the north-west of C^0 and below or on the 45° degree line represents the wealth of the insured with any contract where $\alpha_i \geq 0$ and $\beta_i \geq 0$. Since the monopoly solution implies no-consumer surplus, it must lie on each risk type indifference curve passing through C^0. These indifference curves are strictly convex since $U(\cdot)$ is strictly concave by assumption[3].

1.2 Private information and single-period contracts

Under private information the insurer does not observe the individual's risk types[4], and must introduce mechanisms to ensure that agents will reveal this characteristic. Stiglitz (1977) extended the Rothschild-Stiglitz (1976) model to the monopoly case. In both contributions, price-quantity contracts[5] permit the separation of risks by introducing incentives for individuals to reveal their type. Low risk individuals reveal their identity by purchasing a policy which offers limited coverage at a low unit price. Thus they trade off insurance

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[3]Since individuals of different types have the same degree of risk aversion, at each point in the figure, the absolute value of the slope of the high risk indifference curve is lower than that of the low risk individual. For example at point C^0, $U'(W)(1 - p_H)/U'(W - D)p_H < U'(W)(1 - p_L)/U'(W - D)p_L$. At equilibrium points C_H^* and C_L^*, the respective slopes (in absolute values) are $(1 - p_H)/p_H$ and $(1 - p_L)/p_L$. This is true since under full insurance, the insured of type i has $W - p_iD - z_i^*$ in each state.

[4]For models where neither the insurer nor the insured know individuals' probabilities of accident see Palfrey and Spatt (1985), Malueg (1988) and Boyer, Dionne and Kihlstrom (1989).

[5]We limit our discussion to private market mechanisms. On public provision of insurance and adverse selection see Pauly (1974) and Dahlby (1981).

protection to signal their identity. Formally, risk revelation is obtained by adding two self-selection constraints to Problem 1 :

$$V(C_i|p_i) - V(C_j|p_i) \geq 0 \qquad \begin{matrix} i = H,L \\ j = H,L \\ i \neq j \end{matrix} \qquad (3)$$

Equation (3) guaranties that individual i prefers C_i to C_j. Let us use λ_{HL} and λ_{LH} for the corresponding Lagrangian multipliers where λ_{HL} is for the self-selection constraint of the H type risk. λ_{HL} and λ_{LH} cannot both be positive[6]. From Figure 1 it is easy to observe that, if the high risk individuals are indifferent between both contracts ($\lambda_{HL} > 0$), the low risk individuals will strictly prefer their own contracts ($\lambda_{LH} = 0$). Moreover, λ_{LH} cannot be positive when λ_{HL} is zero since this leads to a violation of (2). Therefore, a feasible solution can be obtained only when $\lambda_{HL} > 0$ and $\lambda_{LH} = 0$.

Figure 1 shows the solution to the maximization of (1) subject to (2) and (3) where low risk individuals choose a positive quantity of insurance[7] $\beta_L^{**} > 0$ and high risk individuals buy full insurance coverage ($\beta_H^{**} = \beta_H^{*}$). Separation of risks and profit maximization imply that $V(C_H^{**}|p_H) = V(C_L^{**}|p_H)$. As discussed above, it is clear that (3) and (2) cannot both be binding for the high risk individuals when it is possible for the low risks to buy insurance. In fact, Figure 1 indicates that C_H^{**} is strictly preferred to C_H^{*} which means that high risk individuals get some consumer surplus when the monopolist sells insurance to the low risk individuals. In other words, the rationality constraint (2) is not binding for the H individuals ($\lambda_H = 0$).

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[6]Technically the preference structure of the model implies that indifference curves of individuals with different risks cross only once. This single crossing property has been used often in the sorting literature (Cooper, 1984).

[7]It is important to note that there is always a separating equilibrium in the monopoly case. However, the good risk individuals may not have any insurance coverage at the equilibrium. Property 4 in Stiglitz (1977) establishes that $C_L^{**} = 0$ when q_H/q_L exceeds a critical ratio of high to low risk individuals. The magnitude of the critical ratio is function of the difference in accident probabilities and of the size of the damage. Here, in order to have $C_L^{**} > 0$, we assume that q_H/q_L is below the critical ratio.

Another property of the solution is that good risk individuals do not receive any consumer surplus ($\lambda_L > 0$). However, as discussed above, they strictly prefer their contract to the contract offered to the bad risk individuals. In other words

$$V(C_L^{**}|p_L) = V(C^0|p_L) \text{ and } V(C_L^{**}|p_L) > V(C_H^{**}|p_L)$$

which means that the self-selection constraint is not binding for the low risk individuals while the rationality constraint is.

In conclusion, one-period contracts with a self-selection mechanism increase the monopoly profits under private information compared with a single contract without any revelation mechanism, but do not necessarily correspond to the best risk allocation arrangement under asymmetrical information. In particular, good risk individuals may not be able to buy any insurance coverage or, if they can, they are restricted to partial insurance. As we shall see in the next section, multi-period contracts can be used to relax the binding constraints and to improve resource allocation under asymmetrical information. In summary

Proposition 2 : In the presence of private information, an optimal one-period contract menu between a private monopoly and individuals of types H and L has the following characteristics :

a) $\beta_H^{**} = D - \alpha_H^{**}$; $\beta_L^{**} < D - \alpha_L^{**}$
b) $V(C_H^{**}|p_H) > V(C^0|p_H)$; $V(C_L^{**}|p_L) = V(C^0|p_L)$
c) $V(C_H^{**}|p_H) = V(C_L^{**}|p_H)$; $V(C_L^{**}|p_L) > V(C_H^{**}|p_L)$

Proof : See Stiglitz (1977).

Stiglitz (1977) also considered a continuum of agent types and showed that some of the above results can be obtained under additional conditions. However, in general, the presence of a continuum of agent types affects the results. For example, Riley (1979) showed that a Nash equilibrium never exists in the continuum case. (See also Riley, 1985).

1.3 Multi-period insurance contracts

Multi-period contracts are often observed in different markets. For example, in many countries, drivers buy automobile insurance with the same insurer for many years and insurers use bonus-malus systems (or experience rating) in order to relate insurance premiums to the individual's past experience. (Lemaire, 1985; Henriet and Rochet, 1984; Hey, 1985; Dionne and Vanasse, 1988). Long term contracting also is observed in labour markets, workers' compensation insurance, service contracts, unemployment insurance and many other markets. The introduction of multi-period contracts in the analysis gives rise to many issues such as the time horizon, discounting, commitment of the parties, myopic behaviour, accident underreporting, renegotiation. These issues are discussed in the following paragraphs.

Multi-period contracts are set, not only to adjust ex-post insurance premiums or insurance coverage to past experience, but also as a sorting device. They can be a complement or a substitute to standard self-selection mechanisms. However, in presence of full commitment, ex-ante risk announcement or risk revelation remains necessary to obtain optimal contracts under adverse selection.

In Cooper and Hayes (1987), multi-period contracts are presented as a complement to one period self-selection constraints. Since imperfect information reduces the monopolist's profits, the latter has an incentive to relax the remaining binding constraints by introducing contracts based on anticipated experience over time. By using price quantity contracts and full commitment in long term contracts, Cooper and Hayes introduce a second instrument to induce self-selection and increase monopoly profits : experience rating increases the cost to high-risks from masquerading as low-risks by exposing them to second-period contingent coverages and premia.

Cooper and Hayes' model opens with a direct extension of the standard one-period contract presented above to a two-period world with full commitment on the terms of the contract. There is no discounting and all agents are able to anticipate the values of the relevant futures variables. In order to increase profits, the monopolist offers contracts in which premiums and coverages in the second period are function of accident history in the

first period. Accidents are public information in their model. The two period contract C_i^2 is defined by :

$$C_i^2 = \{\alpha_i, \beta_i, \alpha_{ia}, \beta_{ia}, \alpha_{in}, \beta_{in}\}$$

where a and n mean "accident" and "no accident" in the first period and where α_{il} and β_{il} (l = a,n) are "contingent" choice variables. Conditional on accident experience, the formal problem consists of maximizing two-period expected profits by choosing C_L^2 and C_H^2 under the following constraints :

$$V(C_i^2|p_i) \geq 2V(C^0|p_i) \qquad (4.1)$$

$$V(C_i^2|p_i) \geq V(C_j^2|p_i) \qquad i = H,L \quad j = H,L \quad i \neq j \qquad (4.2)$$

where $V(C_i^2|p_k) \equiv p_k\, U(W - D + \beta_i) + (1 - p_k)\, U(W - \alpha_i) + p_k[p_k\, U(W - D + \beta_{ia}) + (1 - p_k)\, U(W - \alpha_{ia})] + (1 - p_k)\,[p_k\, U(W - D + \beta_{in}) + (1 - p_k)\, U(W - \alpha_{in})]$

$k = i,j \quad j = H,L$

$i = H,L \quad i \neq j$

The above constraints show that agents are committed to the contract for the two periods. The model does not allow the parties to renegotiate the contract at the end of the first period. Moreover, the principal is committed to a loss related adjustment of the insurance contract in the second period negotiated at the beginning of the first period; the insured is committed, for the second period, to buy the coverage and to pay the premium chosen at the beginning of the first period. It is also interesting to observe from (4) that the decisions concerning insurance coverage in each period depend on the anticipated variations in the premiums over time. In other words, (4) establishes that variations in both premia and coverages in the second period are function of experience in the first period. Using the above model, Cooper and Hayes proved the following result :

Proposition 3 : In the presence of private information, the monopoly increases his profits by offering an optimal two-period contract having the following characteristics :

1) High risk individuals obtain full insurance coverage in each period and are not experience rated,

$$\hat{\alpha}_H = \hat{\alpha}_{1H} = \hat{\alpha}_{2H}, \ \hat{\beta}_H = \hat{\beta}_{2H} = \hat{\beta}_{1H}$$

where $\hat{\beta}_H = D - \hat{\alpha}_H$

2) Low risk individuals obtain partial insurance with experience rating,

$$\hat{\alpha}_{1L} < \hat{\alpha}_L < \hat{\alpha}_{2L}, \ \hat{\beta}_{2L} < \hat{\beta}_L < \hat{\beta}_{1L}$$

3) Low risk individuals do not obtain any consumer surplus and high risk individuals are indifferent between the two contracts,

$$V(\hat{C}_L^2|p_L) = 2V(C^0|p_L),$$

$$V(\hat{C}_H^2|p_H) = V(\hat{C}_L^2|p_H)$$

Proof : See Cooper and Hayes (1987).

The authors also discussed an extension of their two-period model to the case where the length of the contract may be extended to many periods. They showed that the same qualitative results as those in Proposition 3 hold with many periods.

Dionne (1983) and Dionne and Lasserre (1985, 1987) also investigated multi-period contracts in presence of adverse selection[8]. Their models differ from that of Cooper and Hayes in many respects. The main differences concern the revelation mechanism, the sorting device, commitment assumptions and the consideration of statistical information. Moreover, accidents are private information in their models. Unlike Cooper and Hayes, Dionne (1983) did not introduce self-selection constraints in order to obtain risk revelation. Instead risk revelation results from a Stackelberg game where the insurer offers a contract

[8]Townsend (1982) discussed multi-period borrowing-lending schemes. However, his mechanism implies a constant transfer in the last period which is not compatible with insurance in presence of private information.

in which the individual has to select an initial premium by making a risk announcement in the first period. Any agent who claims to be a low risk pays a corresponding low premium as long as his average loss is less than the expected loss given his declaration (plus a statistical margin of error to which we shall return). If that condition is not met, he is offered a penalty premium. Over time, the insurer records the agent's claims and offers to reinstate the policy at the low premium whenever the claims frequency become reasonable again[9].

Following Dionne (1983) and Dionne and Lasserre (1985), the no-claims discount strategy consists of offering two full insurance premiums[10] ($F^1 = \{\alpha_L, \alpha_H\}$) in the first period and for t = 1,2, ...

$$F^{t+1} \begin{cases} = \alpha_d \text{ if } \sum_{s=1}^{N(t)} \theta^s/N(t) < E_d D(x) + \delta_d^{N(t)} \\ = \alpha_k \text{ otherwise} \end{cases}$$

where

α_d is the full information premium corresponding to the declaration (d), $d \varepsilon \{L,H\}$

θ^s is the amount of loss in contract period s, $\theta^s \varepsilon \{0,D\}$

α_k is a penalty premium. α_k is such that $U(S - \alpha_k) < V(C_0|p_H)$

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[9]This type of "no-claims discount" strategy was first proposed by Radner (1981) and Rubinstein and Yaari (1983) for the problem of moral hazard (see also Malueg (1986) where the "good faith" strategy is employed). However, since the two problems of information differ significantly the models are not identical. First the information here does not concern the action of the agent (moral hazard) but the type of risk which he represents (adverse selection). Second, since the action of the insured does not affect the random events, the sequence of damage levels is not controlled by the insured. The damage function depends only on the risk type. Third, in the adverse selection model, the insured cannot change his declaration and therefore cannot depart from his initial risk announcement although he can always cancel his contract. Therefore, the stronger conditions used by Radner (1981a) (robust epsilon equilibrium) and Rubinstein and Yaari (1983) ("long proof") are not needed to obtain the desired results in presence of adverse selection only. The Law of the Iterated logarithm is sufficient.

[10]In fact their formal analysis is with a continuum of risk types.

$E_dD(x)$ is the expected loss corresponding to the announcement (d)

$\delta_d^{N(t)}$ is the statistical margin of error

$N(t)$ is the total number of periods with insurance; $N(t) \leq t$.

Therefore, from the construction of the model, $\sum_{s=1}^{N(t)} \theta^s/N(t)$ is the average loss claimed by the insured in the first N(t) periods. If this number is strictly less then the declared expected loss plus some margin of error, the insurer offers α_d. Otherwise he offers α_x. The statistical margin of error is used in order not to penalize too often those who tell the truth. But it has to be small enough to detect those who try to increase their utility in announcing a risk class inferior to their true risk. From the Law of the Iterated Logarithm, one can show that

$$\delta_d^{N(t)} = \sqrt{2\gamma\sigma_d^2 \log \log N(t)/N(t)}, \qquad \gamma > 1$$

where σ_d^2 is the variance of the individual's loss corresponding to the declaration (d) and $\delta_d^{N(t)}$ converges to zero over time (with arbitrary large values for N(t) = 1,2)

Graphically, we can represent $E_dD(x) + \delta_d^{N(t)}$ in the following way :

Figure 2 here

$$\text{As } N(t) \to \infty,\ E_dD(x) + \delta_d^{N(t)} \to E_dD(x)\ .$$

Over time, only a finite number of points representing ($\Sigma\ \theta^s/N(t)$) will have a value outside the shaded zone.

Proposition 4 below shows that the public information allocation of risks is obtainable using the no-claims discount strategy as $T \to \infty$ and as long as the agents do not discount the future[11].

Proposition 4 : Let α_i be such that :

$$\alpha_i - E_iD(x) \geq 0 \text{ and } U(W - \alpha_i) \geq V(C^0|p_i) .$$

Then, when $T \to \infty$, there exists a pair of optimal strategies for the individual of type i and the private monopoly having the following properties :

1) the strategy of the monopoly is a "no-claims discount strategy"; the strategy of insured i is to tell the truth about his type in period 1 and to buy insurance in each period;

2) the optimal corresponding payoffs are $\alpha_i^* - E_iD(x) = z_i^*$ and $U(W - \alpha_i^*) = V(C^0|p_i)$, i = H,L;

3) both strategies are enforceable.

Proof : See Dionne and Lasserre (1985).

It is also possible to obtain a solution close to the public information allocation of risks in finite horizon insurance contracts. Dionne and Lasserre (1987) showed how a trigger

[11] In general, introducing discounting in repeated games reduces the incentives of telling the truth and introduces some inefficiency because players do not care for the future as they care for the current period. In other words, with discounting, players become less patient and cooperation becomes more difficult to obtain. See Sabourian (1989) and Abreu, Pearce and Stacchetti (1990) for detailed discussions on the discount factor issues in repeated contracts. See also footnote 15.

strategy with revisions[12] may establish the existence of an ε-equilibrium. This concept of ε-equilibrium is due to Radner (1981) and was also developed in a moral hazard context. Extending the definition to the adverse selection problem, Dionne and Lasserre (1987) defined an ε-equilibrium as a triplet of strategies (principal, low risk individual, high risk individual) such that, under these strategies, the expected utility of any one agent is at least equal to his expected utility under public information less epsilon. In fact, the expected utility of the high risk individual is that of the full information equilibrium.

As for the case of an infinite number of periods[13], Dionne and Lasserre (1987) showed that it is in the interest of the monopolist (he obtains higher profits) to seek risk revelations on the part of the insured rather than simply use the statistical instrument to discriminate between low-risk and high-risk agents. In other words, their second main result shows that it is optimal to use statistical tools not only to adjust, ex-post, insurance premiums according to past experience, but also, to provide an incentive for the insured to announce, ex-ante, the true class of risk he represents. Finally, they obtained that a multi-period contract with announcement dominates a repetition of one-period self-selection mechanisms (Stiglitz, 1977) when the number of periods is sufficiently large and there is no discounting.

Another characteristic of Dionne and Lasserre (1987) model is that low risk agents do not have complete insurance coverage when the number of periods is finite; they chose not to insure if they are unlucky enough to be considered as high risk individuals. However, they always choose to be insured in the first period and most of them will obtain full insurance in each period. Finally, it must be pointed out that the introduction of a continuum of agent types does not create any difficulty in the sense that full separation of risks is obtained without any additional condition.

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[12]Radner's (1981) contribution does not allow for revisions after the initial trigger. However, revisions were always present in infinite horizon models [Rubinstein and Yaari (1983), Dionne (1983), Radner (1985), Dionne and Lasserre (1985)]. A trigger strategy without revision consists of offering a premium corresponding to a risk declaration as long as the average loss is less than the reasonable average loss corresponding to the declaration. If that condition is not met, a penalty premium is offered for the remaining number of periods. With revisions, the initial policy can be reinstate.

[13]See also Gal and Landsberger (1988) on small sample properties of experience rating insurance contracts in presence of adverse selection. In their model, all insureds buy the same contracts and resort to experience is made in the premium structure only. They show that the monopoly's expected profits are higher if based on contracts which take advantage of longer experience.

In Dionne (1983) and Dionne and Lasserre (1985) there is no incentive for accidents underreporting at equilibrium since there is no benefit associated with underreporting. When the true classes of risk are announced, insureds cannot obtain any premium reduction by underreporting accidents. When the number of periods is finite, matters are less simple since each period does matter. In some circumstances, the insured has to evaluate the trade-off between increased premiums in the future and no coverage in the present. This is true even when the contract involves commitment and no renegotiation as in Dionne and Lasserre (1987). For example, the unlucky good risk may prefer to receive no insurance coverage during a particular period in order to pass over a trigger date and have the opportunity to pay the full information premium as long as his average loss is less than the reasonable average loss corresponding to his class of risk.

We next address the incentive for policyholders to underreport accidents. The benefits of underreporting can be shown to be nil in a two-period model with full commitment and no statistical instrument and when the contract cannot be renegotiated over time (see the Appendix for a formalization of the following discussion). To see this, let us go back to the two-period model presented earlier (Cooper and Hayes, 1987) and assume that accidents are now private information. When there is ex ante full commitment by the two parties to the contract one can write a contract where the net benefit to any type of agent from underreporting is zero. High risk individuals have full insurance and no experience rating at equilibrium and low risk have the same level of expected utility whatever the accident reporting at the end of the second period. However, private information about accidents reduces insurer's profits when we compare with the situation where accidents are public information.

In all the preceding discussions it was assumed that the insurer can precommit to the contract over time. It was shown that an optimal contract under full commitment can be interpreted as a single transaction where the incentive constraints are modified to improve insurance possibilities for the low risk individuals and to increase profits. Since there is full commitment and no renegotiation, accident histories are uninformative on the risk type. This form of commitment is optimal in Dionne (1983) and Dionne and Lasserre (1985) since, as in the Arrow-Debreu world, neither party to the contract can gain from renegotiation. However, in a finite horizon world, the role of renegotiation becomes important since self-

selection in the first period implies that future contracts might be inefficient given the public information available after the initial period. When the good risks have completely revealed their type, it becomes advantageous to both parties, the insurer and the low risk individuals, to renegotiate a full insurance contract for the second period. Although the possibilities of renegotiation improve welfare in the second period, they violate the ex-ante self-selection constraints and reduce ex-ante welfare. In other words, renegotiation limits the commitment possibilities and reduces ex-ante parties welfare. For example, if the high risk individuals anticipe renegotiations in the second period, they will not reveal their type in the first period (Dionne and Doherty, 1991).

Formally, we can interpret the possibility of renegotiation as adding a new constraint to the set of feasible contracts : unless parties can precommit not to renegotiate then contracts must be incentive compatible and renegotiation-proof (Dewatripont, 1989; Bolton, 1990). In order to reduce the possibilities for renegotiation in the second period, the insurer who is unable to commit not to renegotiate after new information is revealed, must set the contracts so that the insured type will not be perfectly known after the first period. This implies that the prospect of renegotiation reduces the speed of information revelation over time. In other words, the prospect of renegotiation can never improve the long term contract possibilities. In many circumstances, a sequence of one period contracts will give the same outcome as a renegotiated-proof long term contract; in other circumstances a renegotiation-proof long term contract dominates (when intertemporal transfers are allowed, for example) (Hart and Tirole, 1988; Laffont-Tirole, 1987, 1990; Dionne and Doherty 1991).

Recently, Hosios and Peters (1989) presented a formal model that rules out any renegotiation by assuming that only one-period contracts are enforceable.[14] They also discussed the possibility of renegotiation in the second period when this renegotiation is beneficial to both parties. Although they cannot show formally the nature of the equilibrium under this alternative, they obtained interesting qualitative results. For example, when the equilibrium contract corresponds to incomplete risk revelation in the first period, the seller offers, in the second period, a choice of contract that depends on the experience of the first period. Therefore accident underreporting is possible with commitment and renegotiation.

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[14]On limited commitment see also Freixas, Guesnerie and Tirole (1985).

This result is similar to that obtained in their formal model where they ruled out any form of commitment for contracts that last for more than one period. Only one-period contracts are enforceable. They showed the following results.

Proposition 5 : In absence of any form of commitment from both parties to the contract :

1) Without discounting, separating equilibria do not exist; only pooling and semiseparating equilibria are possible.[15]

2) Accident underreporting can now affect the seller's posterior beliefs about risk types and insurance buyers may fail to report accidents in order to avoid premium increases.

Proof : See Hosios and Peters (1989).

This first result implies that the insurer does not have full information on the risk types at the end of the first period; therefore, accidents reports become informative on the risk type contrary to the Cooper and Hayes model. However, the authors did not discuss the optimality of such two-period contract. It is not clear that a sequence of one period contracts with separating equilibrium does not dominate their sequence of contracts.

2. COMPETITIVE CONTRACTS

The introduction of competition raises many new issues. The two main issues that will be discussed here are : 1) the choice of an adequate equilibrium concept and 2) the nature of information between competitive insurers. It will be shown that many well known and standard results are sustained under the strong assumptions that all the insurers share

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[15]However, separating equilibria are possible with discounting since future considerations are less relevant. In a model with commitment and renegotiation, Dionne and Doherty (1991) obtain a similar result : when the discount factor is very low a separating equilibrium is always optimal in a two-period framework. Intuitively, low discount factors reduce the efficiency of using intertemporal transfers or rents to increase the optimal insurance coverage of the low risk individuals by pooling in the first period. See Laffont and Tirole (1990) for a general discussion on the effect of discounting on optimal solutions in procurement when there is no uncertainty.

the same information about individuals' choice of contracts and accident experience. In order to discuss these and other issues clearly, let us first present the standard public information case.

2.1 Public information about individuals' characteristics

Under competition, firms are now constrained to earn zero expected profits. When information on individual risk characteristics is public, each firms knows the risk type of each individual. The optimal individual contract is the solution to :

Problem 2 $\underset{\alpha_i,\beta_i,\lambda_i}{\text{Max}}\ p_i\, U(W - D + \beta_i) + (1 - p_i)\, U(W - \alpha_i) + \lambda_i[(1 - p_i)\alpha_i - p_i\beta_i]$

$i = L,H$

where $(1 - p_i)\,\alpha_i - p_i\beta_i$ is the zero profit constraint.

As for the monopoly case under public information, the solution to Problem 2 yields full insurance coverage for each type of risk. The optimal solutions, $\bar{C}_H$ and $\bar{C}_L$ in Figure 3 correspond to levels of consumers' welfare greater than in the no insurance case (C^0). However, as already pointed out, the monopoly solution under private information also yields full insurance coverage and does not introduce any distortion in risk allocation. The difference between the monopoly and competitive cases is that, in the former, consumer surplus is extracted by the insurer, while in the latter it is retained by both types of policyholder.

(Figure 3, here)

Under competition, the zero profit constraint passes through C^0 and the absolute value of its slope is equal to $(1 - p_i)/p_i$. Each point $\bar{C}_i$ represents the expected wealth of an individual of type i. The full information solutions are obtained when the slopes of indifference curves are just equal to the ratio of the probability of not having an accident to that of having an accident.

2.2 Private information and single-period contracts

It is clear that, under asymmetrical information, traditional full information competitive contracts are not adequate to allocate risk optimally. Consequently, many authors have investigated the role of sorting devices in a competitive environment. The first contributions on the subject in competitive markets are by Spence (1973), Rothschild and Stiglitz (1976) and Wilson (1977). The literature on competition in markets with adverse selection is now very large and it is not our intention here to review all significant contributions. Our selection of models was made with criteria that will be identified and explained when it will become appropriate[16].

A first division that we can make is between models of signalling (informed agents move first) and of screening (uninformed agents move first) (Sliglitz and Weiss, 1984). Spence (1973) and Cho and Kreps (1987) models are of the first type and are mainly applied to labour contracts where the workers (informed agents) move first by choosing an education level (signal). Then employers bid for the services of the workers and the latter choose the more appropriate or preferred bids. Cho and Kreps (1987) present conditions under which this three-stage game generates a Riley (1979a) single period separating equilibrium[17]. Without restrictions (or conditions as those proposed by Cho and Kreps (1987)) on out-of-

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[16]See Cresta (1984) and Eisen (1989) for other analyses of problems of equilibria with asymmetric information.

[17]A Riley or "reactive" equilibrium leads the Rothschild-Stiglitz separating equilibrium regardless of the number of individuals in each class of risk.

equilibrium beliefs, many equilibria arise simultaneously which limits considerably the explanatory power of the traditional signalling models[18].

Although it may be possible to find interpretations of the signalling models in insurance markets, it is generally believed that the screening interpretation is more natural. Rothschild and Stiglitz (1976) and Wilson (1977) introduced to the literature an insurance model with a screening behaviour. In their model only a two-stage game is considered. First, the uninformed insurer offers some contracts to the informed agents who then choose among the contracts in the second stage.

Let us start with Rothschild and Stiglitz (1976) where the insurers set premiums with constant marginal costs (as in the Bertrand model). Each insurer knows the proportions of good risks and bad risks in the market but has no information on an insured's type. Moreover, each insured cannot, by assumption, buy insurance from many insurers. Otherwise, the individual insurers would not be able to observe the individual's total amount of insurance coverage and would not be able to discriminate easily[19].

It is clear that the nature of the equilibrium is a function of how individual firms anticipate the behaviour of rivals. Rothschild and Stiglitz (1976) assumed that each insurer follows a pure Cournot-Nash strategy, where each insurer takes the actions of its competitors as given. This results in an equilibrium having the following properties : (a) no contract in the equilibrium set makes negative expected profits; (b) there is no contract outside the equilibrium set that can make a positive profit when included in the original set. They obtained two significant results :

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[18]In fact, multiple equilibria are the rule in two-stage signalling models. However, when the stability of such equilibria are studied the problem is to find at least one that is stable. For a more detailed analysis of signalling models see the survey by Kreps (1989). On the notion of sequential equilibrium and on the importance of consistency in beliefs see Kreps and Wilson (1982).

[19]Jaynes (1978) and Hellwig (1988) analyzed the consequences of relaxing this assumption. More particularly they showed under what conditions an equilibrium exists when the sharing of information about customers is treated endogenously as part of the game among firms. They showed that it is possible to overcome the Rothschild-Stiglitz's existence problem of an equilibrium if insurers compete on price-quantity contracts and exchange information so that insureds cannot buy more than one contract. Finally, Hellwig (1988) showed that the resulting equilibrium is more "akin" to the Wilson reactive equilibrium than to the competitive Nash equilibrium.

Proposition 6 : When insurers follow a pure Cournot-Nash strategy in a two stage screening game :

a) A pooling equilibrium is not possible; the only possible equilibria are separating equilibria.

b) A separating equilibrium may not exist.

A pooling equilibrium is an equilibrium where both types of risk buy the same contract. Assume that the publicly observable proportions of good risk and bad risk individuals are respectively δ and $(1 - \delta)$, and the average price of insurance is $\bar{p}$. This corresponds to the line EF in Figure 4a below. To see why the Cournot-Nash definition of equilibrium is not compatible with a pooling equilibrium, assume that (C_1) in the figure below is a pooling equilibrium contract for a given insurer. By definition, it corresponds to zero expected profits; otherwise another insurer in the market will offer another pooling contract. Because of the relative slopes of the risk type indifference curves, there always exists a contract (C_2) that will be preferred to contract (C_1) by the low risk individuals. The existence of contract (C_2) contradicts the above definition of a Cournot-Nash equilibrium. Consequently, if there exists an equilibrium it has to be a separating one where different risk-type consumers receive different insurance contracts. As for the monopoly case, the formal solution is obtained by adding one self-selection constraint to Problem 2 and by verifying that, at equilibrium the high risk individuals receive full insurance and the low risk types receive less than full insurance.[20] (See Wilson (1977) for a formal analysis). Again the self-selection constraint is binding for the high risk individuals only and the profit constraint is binding at zero expected profits.

(Figure 4, here)

[20]Partial coverage is generally interpreted as a monetary deductible. However, in many insurance markets the insurance coverage is excluded during a probationary period which can be interpreted as a sorting devise. Fluet (1990) analysed the selection of an optimal time-deductible in presence of adverse selection.

Graphically, $\hat{C}_L$ and $\hat{C}_H$ in Figure 4b correspond to a separating equilibrium. In equilibrium, high risk individuals buy full insurance $(\hat{C}_H - \bar{C}_H)$, while low-risk individuals get only partial insurance $(\hat{C}_L)$ [21]. Each firm earns zero expected profit on each contract. This equilibrium has the advantage for the low risk agents that their equilibrium premium corresponds to their actuarial risk and does not contain any subsidy to the high risk individuals. However, a cost is borne by low risk insureds in that their equilibrium contract delivers only partial insurance compared with full insurance in the full information case. Only high risk individuals receive the first best risk allocation. Finally, the separating equilibrium may not be Pareto optimal in the sense that it is possible to improve the welfare of individuals in each class of risk. We will come back to this issue.

The second important result from Rothschild and Stiglitz is that there are conditions under which a separating equilibrium does not exist. In general, there is no equilibrium if the costs of pooling are low to the low risk individuals (few high risk individuals or low $(1 - \delta)$ which is not the case in Figure 4b since the line C^0F^1 corresponds to a value of δ lower than the critical level δ^{RS} permitting separating equilibria) or if the costs of separating are high (structure of preferences). In the former case, given the separating contracts, the cost of sorting (partial insurance) exceeds the benefits (no subsidy) when profitable pooling opportunities exist. But, as already shown, a pooling contract cannot be an equilibrium. This negative result has prompted further theoretical investigations since many insurance markets do function even in presence of adverse selection.

One such extension is to consider a mixed strategy where an insurer's strategy is a probability distribution over a pair of contracts. Rosenthal and Weiss (1984) showed that

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[21] On the relationship between the coverage obtained by a low-risk individual under monopoly compared to that under the pure Nash competitive equilibrium, see Dahlby (1987). It is shown for example, that under constant absolute risk aversion, the coverage obtained by a low risk individual under monopoly is greater than, equal to, or less than, that obtained under competition as the monopolist's expected profit on a policy purchased by low-risk individuals is greater than, equal to, or less than, its expected profit on the policy purchased by high risk individuals.

a separating Nash equilibrium always exists when the insurers adopt this type of strategy.[22] However, it is not clear that a mixed strategy has any particular economic interpretation in insurance markets as in many other markets. Another extension is to introduce a three-stage game where the insurer may reject in the third stage, the insured's contract choice made in the second stage. Hellwig (1986, 1987) showed that a pooling contract may correspond to a sequential equilibrium of the three-stage game or can never be upset by a separating contract whenever a pooling is Pareto preferred. Moreover, contrary to the Rothschild-Stiglitz two-stage model, the three stage game always has a sequential equilibrium in pure strategies. The most plausible sequential equilibrium is pooling rather than sorting while in a three-stage game in signalling models (Cho and Kreps, 1987) it is the pooling rather than the separating equilibria that lack robustness. As pointed out by Hellwig (1987), the conclusions are very sensitive to the details of game specification.

Another type of extension that permits the existence of equilibria is to allow firms to consider other firms' behaviour or reactions in their strategies and then to abandon the pure Nash strategy in the two-stage game. For example, Wilson (1977) proposed an "anticipatory" equilibrium concept where firms drop policies so that those remaining (after other firms anticipated reactions) at least break even. The resulting equilibrium (pooling or separation) always exists. By definition, a Wilson equilibrium exists if no insurer can offer a policy such that : 1) this new policy yields nonnegative profits and 2) remains profitable after other insurers have withdrawn all nonprofitable policies in reaction to the offer.

A Wilson equilibrium is a Nash equilibrium when a separating equilibrium exists; otherwise it is a pooling equilibrium.[23] Wilson also considered subsidization between policies, but Miyazaki (1977) and Spence (1977) developed the idea more fully. They showed how to improve welfare of both classes of risk (or of all n classes of risk; Spence (1977)) with low-risk class subsidizing the high-risk class. In fact, Spence has shown that,

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[22]See also Dasgupta and Maskin (1986). Mixed strategy and randomization are not equivalent. On randomization to improve market functioning in presence of adverse selection see Garella (1989) and Arnott and Stiglitz (1988).

[23]As pointed above the Riley (1979) "reactive" equilibrium is also a relevant alternative to the Nash equilibrium. See Grossman (1979) for an analysis of the Wilson type equilibrium with reactions of insureds rather than reactions of sellers.

in a model where firms react (in the sense of Wilson) by dropping loss-making policies, there is a unique equilibrium. In all the above models, each of the contracts in the menu available is defined to permit the low risk policyholders to signal their true risk. The resulting equilibrium is a break-even portfolio of separating contracts and exists regardless of the value of δ. More formally we have :

Proposition 7 : A Wilson-Miyazaki-Spence (WMS) equilibrium exists regardless of the value of δ. When $\delta \leq \delta^{RS}$ the WMS equilibrium corresponds to the Rothschild-Stiglitz equilibrium.

One such equilibrium (C_3, C_4) is presented in Figure 5 below for the case of two risk classes with cross-subsidization from the low to the high risk group. The line (C^0 C_3 Z) is the zero profit locus defined such that contract pairs yield a zero profit to the insurer.

(Figure 5, here)

The above equilibrium is second-best efficient in the sense of Harris and Townsend (1981). An allocation is second best efficient if it is Pareto optimal within the set of allocations that are feasible under the feasible constraints (self-selection constraints and the zero profit constraints on the portfolio)[24]. In fact Crocker and Snow (1985) proved the following proposition :

Proposition 8 : A Wilson-Miyazaki-Spence equilibrium is second-best efficient for all values of δ.

Proof : See Crocker and Snow (1985).

It can be shown that a Rothschild-Stiglitz equilibrium is second-best efficient if and only if δ is lower than some critical value $\hat{\delta}$ which is itself lower than the critical value δ^{RS} permitting the existence of a Nash equilibrium. Then, as mentioned above, a Nash

[24]See Crocker and Snow (1985, 1986) for more details.

equilibrium is not necessarily efficient. The same conclusion applies to the Riley equilibrium since it sustains the Rothschild-Stiglitz solution whatever the value of δ. Finally, as we will see in the next section, another possibility to deal with equilibrium issues is to use risk categorization (Bond and Crocker, 1990).

2.3. Multi-period contracts

Let us begin with Cooper and Hayes (1987) analysis of two-period contracts with full commitment on the supply side. Cooper and Hayes used the Nash equilibrium concept in a two-stage, two-period game which implies that the equilibrium must be a separating equilibrium.[25] In fact, they considered two different behaviours about commitment. First, both insurers and insureds commit themselves to the two-period contracts (without renegotiation) and, second, the insurers commit to a two period contract but the contract is not binding on insureds. We will refer the respective assumptions as contracts with full commitment and with semi-commitment. When competitive firms can bind agents to the two periods, it is easy to show that, in the separating solution, the contracts offered are qualitatively identical to that of the monopoly solution with commitment : high-risk agents receive full insurance at an actuarial price in each period while low risk agents face price and quantity adjustments in the second period.

When the authors relax the strong commitment assumption in favor of semi commitment, and consider that insureds can costlessly switch to other firms in the second period, they show that the presence of second-period competition limits, but does not destroy, the use of experience rating as a sorting device. The difference between the results with full commitment and semi commitment is explained by the fact that the punishment possibilities for period one accidents are reduced by the presence of other firms that offer single-period contracts in the second period.

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[25] In other words, they implicitly assumed that the conditions to obtain a Nash separating equilibrium in a single period contract are sufficient for their two-period model.

The semi commitment result was obtained by assuming that in the second period, entrant firms offer single-period contracts without any knowledge of insureds' accident histories or their choice of contract in the first period. The new firms' optimal behaviour is to offer a Rothschild-Stiglitz separating contract to those who switch.[28] By taking this decision as given, the design of the optimal two-period contract by competitive firms with semi-commitment has to take into account at least one supplementary binding constraint which reduces social welfare when we compare to full commitment. In other words, the presence of competition, combined with agents' inability to enforce multi-period contracts, reduces the usefulness of long term contracts as a sorting device and consequently the welfare potential gains of long term relationships. This conclusion is similar to that obtained in the monopoly case where the principal cannot commit on non-renegotiation.

The above results can be summarized by the following proposition which was derived from the maximization problem (A2) presented in the Appendix :

Proposition 9 : Let us assume that a Nash equilibrium exists. Then the optimal two-period contracts are characterized by the following properties :

a) With full commitment

$\alpha_{Ln} < \alpha_L < \alpha_{La}$, $\beta_{La} < \beta_L < \beta_{Ln}$

b) With semi commitment

$\alpha_{Ln} < \alpha_L$, $\beta_L < \beta_{Ln}$

c) In both cases

$\alpha_{Hn} = \alpha_H = \alpha_{Ha}$, $\beta_{Ha} = \beta_H = \beta_{Hn}$

An important result arising from semi commitment concerns the evolution of profits over time. The insurers are constrained to earn zero expected profits in aggregate for the two periods. For high risk individuals, expected profits are zero in each period. At equilibrium, firms make positive expected profits on low risk individuals contracts from the first period contract and from the second period contract corresponding to the loss state in the

[28] It is important to repeat here that the authors limited their focus on separating solutions.

first period. Negative profits are earned on second period contracts for low risks who suffered no first period loss. In aggregate, expected two period profits for low risks are zero. This arrangement provides an appropriate bonus for accident free experience and ensures that low risks who suffer an accident remain with the firm.[27]

This temporal profit pattern for the good risks was labelled "highballing" by D'Arcy and Doherty (1990) and was shown to stand in contrast with the "lowballing" predicted by Kunreuther and Pauly (1985).[28] Nilssen (1990) also obtained a "lowballing" prediction. Despite the clear differences between the last two contributions, the results are quite similar. Insurers make negative expected profits in the first period and make positive profits on the policies they renew. The similarity in this "lowballing" prediction is mainly due to the fact that both articles assume that long term contracts are not written while, as we saw, Cooper and Hayes permitted long term contracting. The three contributions assume that insurers have private information on their own policyholders but Kunreuther and Pauly also assume that insurers have private information on current contracts which does not allow them to use the price-quantity contracting. They argue that insurers are unable to write exclusive contracts; instead they propose that insurers offers only pure price contracts (Pauly, 1974) which implies that they necessarily obtain a pooling equilibrium. Over time, the contracting insurer obtains Bayesian update of its policyholders' loss distribution not available to rivals so that policyholders do not sort themselves into distinct risk classes. In contrast to Kunreuther and Pauly, Nilssen (1990) use the basic framework of the Rothschild-Stiglitz model. This last difference explains the types of equilibrium obtained while their similar assumptions on commitment and observation of individual's accident history explain their similar conclusions on lock-in where each firm earns a positive expected profit on its old customers since it

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[27] In fact, the corresponding expected utility of the low risk individual who did not have an accident in the first period (and stay) is strictly greater at equilibrium to that corresponding to the entrant one-period contract.

[28] In our discussion on Kunreuther and Pauly model we refer to their single-period equilibria where firms can modify their prices from one period to the next and consumers are free to leave or stay.

controls information on past experience.[29] Finally, contrarity to the monopoly case, absence of commitment in Nilssen model does not rule out separation.

The next table compares the competing models :

TABLE 1

Characteristics	Kunreuther and Pauly single period equilibria	Cooper and Hayes non binding model	Nilssen	Hosios and Peters with competition	Dionne and Doherty
Price-quantity contracts	no	yes	yes	yes	yes
Commitment to more than one period					
insurer	no	yes	no	no	yes
insured	no	no	no	no	no
Insurers observe					
total contract choice	no	yes	yes	yes	yes
accidents	no (claims only)	yes	yes	no (claims only)	yes
Rivals observe					
contract choices	no	yes *in the current period no for entrants*	yes	yes	yes
loss experience	no	no	no	yes	no
Type of equilibrium					
first period	pooling	separating	pooling or separating	pooling or separating	pooling or separating
second period	pooling	separating	separating	pooling or separating	separating
Accidents underreporting	?	no	no	yes *when pooling in the second period*	no
Consumer lock-in	yes	no	yes *when pooling in the first period*	?	no
Temporal profit pattern	lowballing	highballing	lowballing	?	highballing when pooling in the first period

[29]Cromb (1990) considered the effects of different pre-commitment assumptions between the parties to the contract on the value of accident history. Under fully-binding contracts, the terms of the contract depend only on the number of accidents over a certain time-horizon while under other assumptions (partially-binding and no-binding) the timing of accidents become important.

The above table presents also some extensions of Hosios-Peters model to competition. As for Kunreuther and Pauly, and Nilssen, Hosios and Peters assume no commitment on the supply side and as Nilssen they consider that rivals observe all contract choices. For Nilssen, accidents are private information for the insured and the insurer and are thus not observed by the rival firms. Accident reporting is public information in Hosios and Peters while accidents are not. This last distinction permits explicit consideration of accidents underreporting, an important empirical phenomenon in the automobile insurance industry. Finally, Dionne and Doherty (1991) introduce renegotiation in a model with commitment. They first show that fully separating strategies as in Cooper and Hayes (1987), when renegotiated proofed, degenerate to replications of single period equilibria. They then present an alternative equilibrium which my involve partial pooling in the first period followed by separation. They show that the equilibrium is a full separating equilibrium when the discount factor is low and tends to a pooling for large discount factors.

The issue of consumer lock-in has motivated D'Arcy and Doherty (1990) to undertake an empirical investigation of the significance of adverse selection and of the testable predictions that permit discrimination between the competing models. Their results first show evidence of adverse selection in the U.S. automobile insurance markets, a result also found by Dahlby (1983, 1990) in the Canadian automobile insurance market. The also show empirical evidence of "lowballing" which is consistent with the Nilssen and Kunreuther and Pauly models. Dionne and Doherty (1991) extended D'Arcy and Doherty (1990) empirical analysis. Most of their results also favor the lowballing predictions.

2.3 Risk categorization

Adverse selection can explain the use of risk categorization in insurance markets based on variables that procure information at a low cost (Hoy, 1982). For example, in automobile insurance, age and sex variables are quite significant in explaining individuals' probabilities of accident (Dionne and Vanasse, 1988). Particularly, young male drivers (less than 25) are much more risky than the average driver. Since it is almost costless to observe age and sex, an insurer may find profitable to offer policies with higher premiums to young males. However, such categorization is now prohibited in some states and countries.

As mentioned above, Dahlby (1983) provided some empirical evidence that adverse selection is present in the Canadian automobile insurance market. He also suggested that his empirical results are in accordance with the Wilson-Miyazaki-Spence model that allows for cross-subsidization between individuals in each segment defined by a categorization variable such as sex or age : low-coverage policies (low risks) subsidizing high-coverage policies (high risks) in each segment.[30] This important statistical result raises the following question. Does statistical categorization enhance efficiency in the presence of adverse selection? In other words, can welfare be improved by using the public information on agents' characteristics (such as age and sex) in offering insurance contracts in presence of adverse selection?

Crocker and Snow (1985, 1986) showed that, if the observable characteristics are correlated with hidden knowledge, costless imperfect categorization always enhances efficiency where efficiency is defined as in Harris and Townsend (1981). However, when categorization entails some positive cost, the results are ambiguous[31].

Another important contribution in Crocker and Snow (1986) concerns the existence of a balanced-budget tax-subsidy policy that provides private incentives to use costless categorization. With appropriate taxes, no agent loses as a result of categorization. The results are shown for the Wilson-Miyazaki-Spence equilibrium concept but can also sustain an efficient allocation in a Nash equilibrium with a tax system (Crocker and Snow, 1985). Finally, their conclusions can be applied to the Wilson anticipatory equilibrium or to the Riley reactive equilibrium, both with a tax system. It then becomes clear that prohibiting discrimination on equity considerations imposes efficiency costs in insurance markets such as automobile insurance where categorization based on age and sex variables is costless.

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[30] However, Riley (1983) argued that the statistical results of Dahlby (1983) are also consistent with both the Wilson (1977) anticipatory equilibrium and the Riley reactive equilibrium (1979). Both models reject cross-subsidization.

[31] On the social value of the acquisition of information by risk categorization in presence of imperfect information, see Doherty (1990), Rea (1990), Hoy (1989) and Crocker and Snow (1990).

3. MORAL HAZARD AND ADVERSE SELECTION

Although in many situations principals face adverse selection and moral hazard problems simultaneously when they design contracts, these two types of asymmetrical information have been given separate treatments so far in the economic literature on risk-sharing agreements. Recently, both information problems have been integrated into a single model where all the parties of the contract are risk neutral (Laffont and Tirole, 1986; Picard, 1987; Caillaud, Guesnerie, Rey and Tirole, 1988; Guesnerie, Picard and Rey, 1988). Although these models involve uncertainty, they are unable to explain arrangements where at least one party is risk averse. In particular they do not apply to insurance. More recently, some authors have attempted to integrate both information problems into a single model where the agent is risk averse[32].

As already discussed by Dionne and Lasserre (1988) such an integration of both information problems is warranted on empirical grounds. Applied studies are still few in this area, but they will find it difficult to avoid considering both kinds of information asymmetry. In the insurance literature, for example, one significant contribution is that by Dahlby (1983) who studied the relationship between statistical discrimination and adverse selection. On the basis of differences in accident frequencies between groups or insurance types, he concluded that adverse selection was present in the market studied. However it is not clear in his study that adverse selection is the only source of resource misallocation since moral hazard might also explain part of the frequency differences (See Dahlby (1990), Arnott (1990), and Dionne and Vanasse (1988) for a similar discussion).

3.1 Monopoly and multi-period contracts

Dionne and Lasserre (1988) showed how it is possible to achieve a first-best allocation of risks when moral hazard and adverse selection problems are present simultaneously. While they draw heavily on the contributions of Rubinstein and Yaari (1983),

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[32]On learning and moral hazard see Malueg (1988). He showed that a simple "good faith" strategy overcomes both the problem of differential learning and moral hazard when payoffs are evaluated by the limit of means.

Dionne (1983) and Dionne and Lasserre (1985), the integration of the two types of information problems is not a straightforward exercise. Since an agent who has made a false announcement may now choose an action that is statistically compatible with his announcement, false announcements may go undetected. They proposed a contract under which the agent cannot profit from this additional degree of freedom. Under a combination of moral hazard and adverse selection, several types of customers can adopt different care levels so that they have identical expected losses. When this happens, it is impossible to distinguish those who produce an efficient level of care from the others on the basis of average losses. However, deviant behaviours can be detected by monitoring deviations from the mean.

Thus the insurer's strategy can be written with more than one simple aggregate (as in Dionne and Lasserre (1985) and Rubinstein and Yaari (1983)). In Dionne and Lasserre (1988) the principal has to monitor two aggregates, the average loss experienced by a given agent and its squared deviation from the mean. However, it was sufficient to get the desired result since in their model the information problem has only two dimensions. More generally, the insurer would have to monitor one moment of the distribution for each hidden dimension.

Combining moral hazard with adverse selection problems in models which use past experience, might involve some synergetic effects. In the model presented in Dionne and Lasserre (1988), the same information required to eliminate either the moral hazard problem alone (Rubinstein and Yaari), or adverse selection alone (Dionne and Lasserre), is used to remove both problems simultaneously. A related subject concerns the efficient use of past information, and the allocation of instruments, toward the solution of each particular information problem. For a long time, self-selection mechanisms have been proposed in response to adverse selection while nonlinear pricing was advocated against moral hazard. In one-period contracts both procedures used separately involve inefficiency (partial insurance) which can be reduced by the introduction of time in the contracts. Dionne and Lasserre showed that self selection may help solve moral hazard problems, as well as adverse selection problems. We will now discuss how the use of two instruments may improve resource allocation and welfare when both problems are present simultaneously in single-period competitive contracts.

3.2 Competitive contracts and risk categorization

One of the arguments often used to justify the prohibition of risk categorization is that it is based on fixed or exogenous characteristics such as, age, race and sex. However, as pointed out by Bond and Crocker (1990), insurers also use other characteristics that are chosen by individuals. They extended Crocker and Snow (1986) previous analysis of risk categorization in presence of adverse selection and examined the equilibrium and efficiency implications of risk categorization based on consumption goods that are statistically related to individual's risks, which they termed "correlative products".

Formally, their model introduces endogenous categorization in an environment characterized by both moral hazard and adverse selection. They show that, while there is a natural tension between the sorting of risk classes engendered by adverse selection and the correction of externalities induced by moral hazard, the use of risk classification improves efficiency in resource allocation. They also obtain that the sorting of risks based on correlative consumption may give a first-best allocation as Nash equilibria when adverse selection is not too severe and when the insurer can observe individual consumption of the hazardous good. This is particularly interesting as an alternative view of how firms, in practice, may overcome the nonexistence of Nash equilibrium problems.

They then considered the case where the insurer cannot observe both the individual's consumption and the individual's characteristics. However, the planner can observe aggregate production of the good. They showed that taxation of the consumption good has now two roles (reduces moral hazard and relaxes self-selection constraints) that permit Pareto improvements.

Cromb (1990) analyzed the simultaneous presence of moral hazard and adverse selection in competitive insurance markets[33] and obtained that the addition of moral hazard to the standard Rothschild-Stiglitz (1976) model with adverse selection has qualitative effects on the nature and existence of equilibrium. Under certain circumstances the addition of moral hazard may eliminate the adverse selection problem but, more generally, it constitutes a new

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[33]See also Whinston for a model of social insurance.

source of non-existence of a Nash equilibrium. This result contrasts with the conclusion obtained by Bond and Crocker (1990) who showed that risk categorization makes Nash equilibrium more likely.

CONCLUSION

In this paper we have reviewed recent developments in the theory of adverse selection in insurance markets. Our objective was to highlight significant results in the literature. We started with the monopoly case. Since the parthheaking article by Stiglitz (1977), many authors have investigated sorting devices other than self-selection in one-period contracts to improve resource allocation under adverse selection. One motivation was that the low risk individuals may not have any insurance coverage at the equilibrium of the single-period contract. Another motivation was that long-term relationships are observed in many insurance markets.

A first important result is that, in presence of full commitment between the parties, multi-period contracts are optimal not only to adjust ex-post insurance premia, but to seek risk revelation on the part of the insured. Multi-period contracts can be a complement (Cooper and Hayes, 1987) or a substitute (Dionne, 1983; Dionne and Lasserre, 1985) to the standard self-selection mechanims used in single-period contracts. However, the role of renegotiation has become important in the recent literature. Under certain circomstances, when the good risks have declared their type in the first period, it becomes advantageous to both parties (the insurer and the good risk individuals) to renegotiate full insurance contracts in the second period. However, although the possibilities of renegotiation improve welfare in the second period, when anticipated they violate the ex-ante self-selection constraints and reduce ex-ante benefits associated to risk revelation. The prospect of renegotiation reduces the speed of information revelation, and pooling or semi-pooling contracts may become optimal (Hosios and Peters, 1989; Dionne and Doherty, 1991).

The introduction of competition raised many issues. The two main issues that were discussed are 1) the choice of an adequate equilibrium concept and 2) the nature of information between insurers. In the case of single-period contract, it was shown by Hellwig (1988) that when the sharing of information about customers is treated endogenously in a two-stage game, the resulted equilibrium is more "akin" to the Wilson reactive equilibrium than to the competitive Nash equilibrium. The predictions about the nature of contracts and the temporal profit patters in multi-period contracting also depend upon the assumptions made on the nature of information between the insurers. Table 1 compares the competing models. It

also compares assumptions made on both commitment between the parties to the contract and information about total contract choice and accident observation.

There was a large literature in the 1970's on the nature of equilibrium concepts in presence of adverse selection. The different contributions were motivated by the negative result obtained by Rothschild and Stiglitz that a Nash equilibrium may not exist in a two-stage game. More recently, three stage games have been investigated in models with asymmetric information. Contrary to the Rothschild and Stiglitz result, it was shown in a model with a screening behavior that the game has always a sequential equilibrium in pure strategies (Hellwig, 1987). Efficiency of different equilibria also was studied. Crocker and Snow (1985) showed, for example, that a Wilson-Miyazaki-Spence equilibrium is second-best efficient in the sence of Harris and Townsend (1981) and that the Rothschild and Stiglitz equilibrium is not necessarily efficient when it exists. The same conclusion applies to the Riley equilibrium.

Adverse selection can explain the use of risk categorization in insurance markets. Crocker and Snow (1986) demonstrated that costless imperfect categorization always enhance efficiency. More recently, Bond and Crocker (1990) extended the above contribution to consider moral hazard and showed that taxation permits Pareto improvements when the planner can observe aggregate production of the consumption goods that are statistically correlated with insurable hazards. But, more importantly, they showed that risk categorization may overcome the nonexistance of Nash equilibrium problems.

Many extensions of the above literature can be considered. In this conclusion we shall restrict ourselves on two of them. The first one concerns the integration of moral hazard and adverse selection into a single model. As mentioned in the last section such extension is warranted on empirical grounds. In many cases, empirical studies are not able to dispense with considering both kinds of information asymmetries simultaneously. The same conclusion applies to private and public interventions that may improve resource allocation in practice. It is also warranted on theoretical grounds since very few contributions have proposed integrations of both problems where at least one party to the contract is risk averse.

The second extension is to propose empirical tests to separate issues on commitment in different lines of insurance. Particularly, it would be interesting to have

answers to the following questions : 1) Is competition in insurance markets strong enough to rule out pooling and semi-pooling contracts in absence of commitment? 2) Is the role of experience rating limited to adjust over time premiums that were originally set in a pooling strategy? 3) Is the presence of moral hazard important enough in different markets to rule out the significance of theoretical predictions that do not consider both problems simultaneously?

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Appendix

A1. Monopoly with private information about accidents

Under private information about accidents, we must add one constraint to the two-period contract proposed by Cooper and Hayes in order to rule out underreporting. In this appendix we show that the following constraint is sufficient :

$$U(W - D + \beta_L) + p_L\, U(W - D + \beta_{La}) + (1 - p_L)\, U(W - \alpha_{La}) \qquad \text{(a.1)}$$
$$- U(W - D - \alpha_L) - p_L\, U(W - D + \beta_{LN}) - (1 - p_L)\, U(W - \alpha_{LN}) = 0 .$$

The above constraint means that the low risk individual is indifferent between accident reporting and accident underreporting when the event occurs in the first period. It reduces the gap between second period levels of expected utility to

$$U(W - D + \beta_L) - U(W - D - \alpha_L) > 0$$

and, therefore, reduces the insurer's possibilities of profits since the latter is now limited in his opportunities to relax the one-period binding constraints.

Maximizing the insurer two-period expected profits by choosing C_L^2 and C_H^2 under constraints (4.1), (4.2) and (a.1) yields as first order conditions for the low risk individual decision variables[34] :

$$\alpha_L : q_L = U'(W - \alpha_L)\left[\lambda_L - \frac{(1 - p_H)}{(1 - p_L)}\,\lambda_{HL}\right] - \frac{\lambda_{LL}}{(1 - p_L)}\,U'(W - D - \alpha_L)$$

[34]Since (a.1) does not contain any H type decision variables, introducing private information about accidents does not affect their optimal values.

$$\beta_L : q_L = U'(W - D + \beta_L)\left[\lambda_L - \frac{p_H}{p_L}\lambda_{HL}\right] + \frac{\lambda_{LL}}{p_L}U'(W - D + \beta_L)$$

$$\alpha_{La} : q_L = U'(W - \alpha_{La})\left[\lambda_L - \frac{(1 - p_H)}{(1 - p_L)}\frac{p_H}{p_L}\lambda_{HL}\right] + \frac{\lambda_{LL}}{(1 - p_L)}U'(W - \alpha_{La})$$

$$\beta_{La} : q_L = U'(W - D + \beta_{La})\left[\lambda_L - \frac{p_H p_H}{p_L p_L}\lambda_{HL}\right] + \frac{\lambda_{LL}}{p_L}U'(W - D + \beta_{La})$$

$$\alpha_{LN} : q_L = U'(W - \alpha_{LN})\left[\lambda_L - \frac{(1 - p_H)(1 - p_H)}{(1 - p_L)(1 - p_L)}\lambda_{HL}\right] - \frac{\lambda_{LL}}{(1 - p_L)}U'(W - \alpha_{LN})$$

$$\beta_{LN} : q_L = U'(W - D + \beta_{LN})\left[\lambda_L - \frac{p_H(1 - p_H)}{p_L(1 - p_L)}\lambda_{HL}\right] - \frac{\lambda_{LL}}{p_L}U'(W - D + \beta_{LN})$$

Under public information on accidents, $\lambda_{LL} \equiv 0$. Solving for α_L^{**} and β_L^{**} we have

$$U'(W - \alpha_L)\left[\lambda_L - \lambda_{HL}\frac{(1 - p_H)}{(1 - p_L)}\right] = U'(W - D + \beta_L)\left[\lambda_L - \frac{p_H}{p_L}\lambda_{HL}\right]$$

which implies that $U'(W - D + \beta_L) > U'(W - \alpha_L)$ and $\beta_L^{**} < D - \alpha_L^{**}$.

Similarly we can verify that $\alpha_{La}^{**} > \alpha_L^{**} > \alpha_{LN}^{**}$ and $\beta_{La}^{**} < \beta_L^{**} < \beta_{LN}^{**}$.

Under private information on accidents, (a.1) is binding and $\lambda_{LL} > 0$. This limits the monopoly's profits possibilities. One can verify that

$$\hat{\alpha}_L > \alpha_L^{\bullet\bullet}, \hat{\beta}_L > \beta_L^{\bullet\bullet}, \hat{\beta}_{La} > \beta_{La}^{\bullet\bullet}, \hat{\alpha}_{La} < \alpha_{La}^{\bullet\bullet}, \hat{\beta}_{La} < \beta_{LN}^{\bullet\bullet}, \hat{\alpha}_{LN} > \alpha_{LN}^{\bullet\bullet},$$

where $\hat{\alpha}$ and $\hat{\beta}$ are optimal values under private information on accident histories.

A2. Two-period problem with competition

The proof here is limited to the low-risk contract.

a) Semi-commitment

Under competition, and semi-commitment $C_L^2 = \{\alpha_L, \beta_L, \alpha_{La}, \beta_{La}, \alpha_{LN}, \beta_{LN}\}$ maximizes $V(C_L^2|p_L)$ subject to

$$2V(C_H^{**}|p_H) \geq V(C_L^2|p_H) \tag{a.2}$$

$$V(C_L^2|p_L) \geq 2V(C_H^{**}|p_L) \tag{a.3}$$

$$V(C_{La}|p_L) \geq V(C_L|p_L) \tag{a.4}$$

$$V(C_{LN}|p_L) \geq V(C_L|p_L) \tag{a.5}$$

$$\begin{aligned}(1 - p_L)\alpha_L - p_L\beta_L + p_L[(1 - p_L)\alpha_{LN} - P_L\beta_{LN}] \\ + (1 - p_L)[(1 - p_L)\alpha_{La} - p_L\beta_{La}] = 0\end{aligned} \tag{a.6}$$

Constraints (a.2) and (a.3) are self-selection constraints where C_H^{**} corresponds to the single-period optimal contract of high-risk individuals and C_L^2 consider the option that high-risk agents can take the single-period contract (C_L) of the new firm (entrant) in the second period (see Cooper and Hayes for more details on C_L^2). Constraint (a.4) and (a.5) take into account the option that low-risk individuals can also switch to another firm in the second period whether he has an accident ($C_{La} = \{\alpha_{La}, \beta_{La}\}$) or not ($C_{LN} = \{\alpha_{LN}, \beta_{LN}\}$) in the first period and (a.6) is the zero expected profit constraint. At the optimal solution constraints (a.2), (a.4) and (a.6) are binding.

b) Full commitment

Under competition and commitment by both parties, the contract is similar to that of the monopoly with commitment which implies that constraints (a.4) and (a.5) do not have any usefulness or economic meaning and that constraint (a.2) does not contain any entrant component. At the optimal solution constraints (a.2)' and (a.6) are binding where (a.2)' is written for (a.2) without any entrant component and is similar to (4.2) in the text.

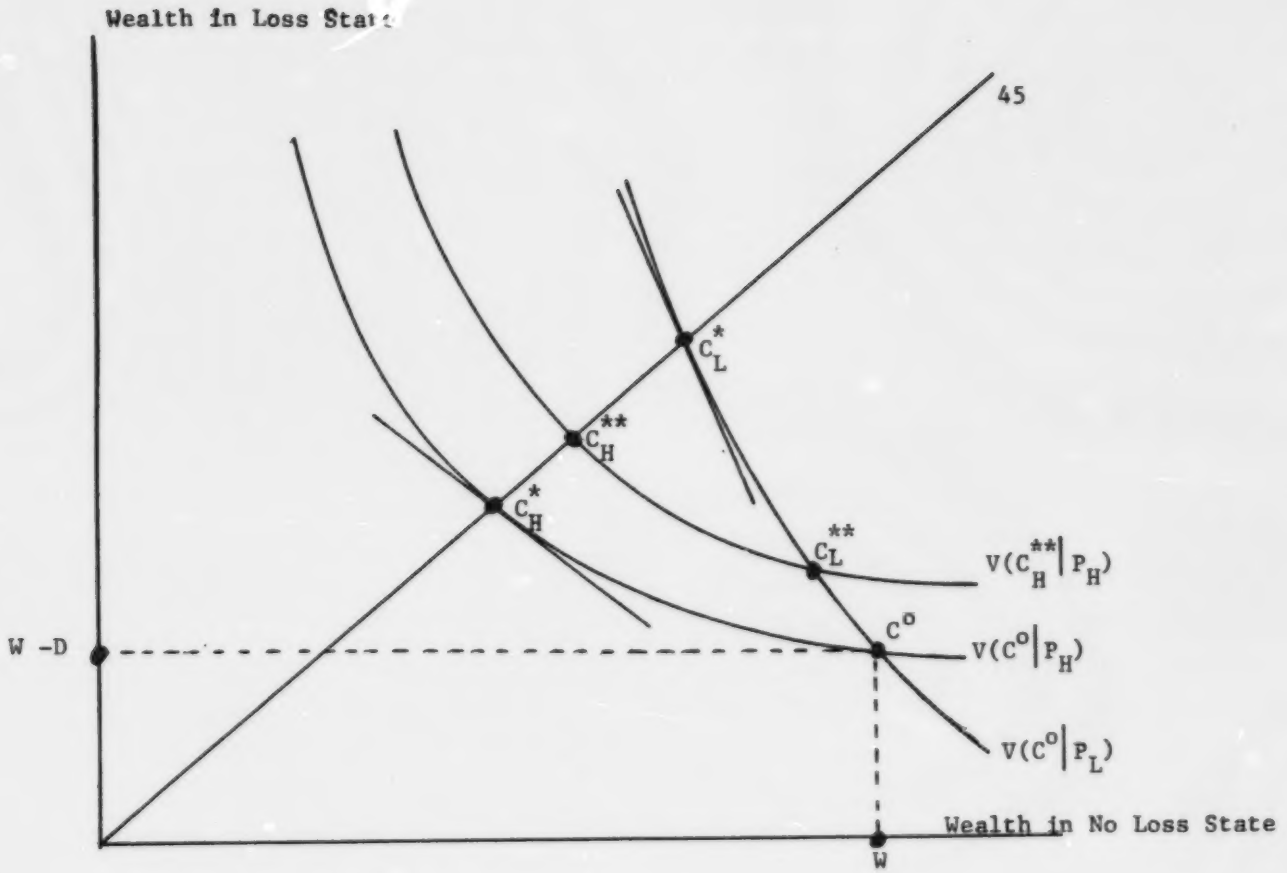


Figure 1 One-Period Monopoly Contract

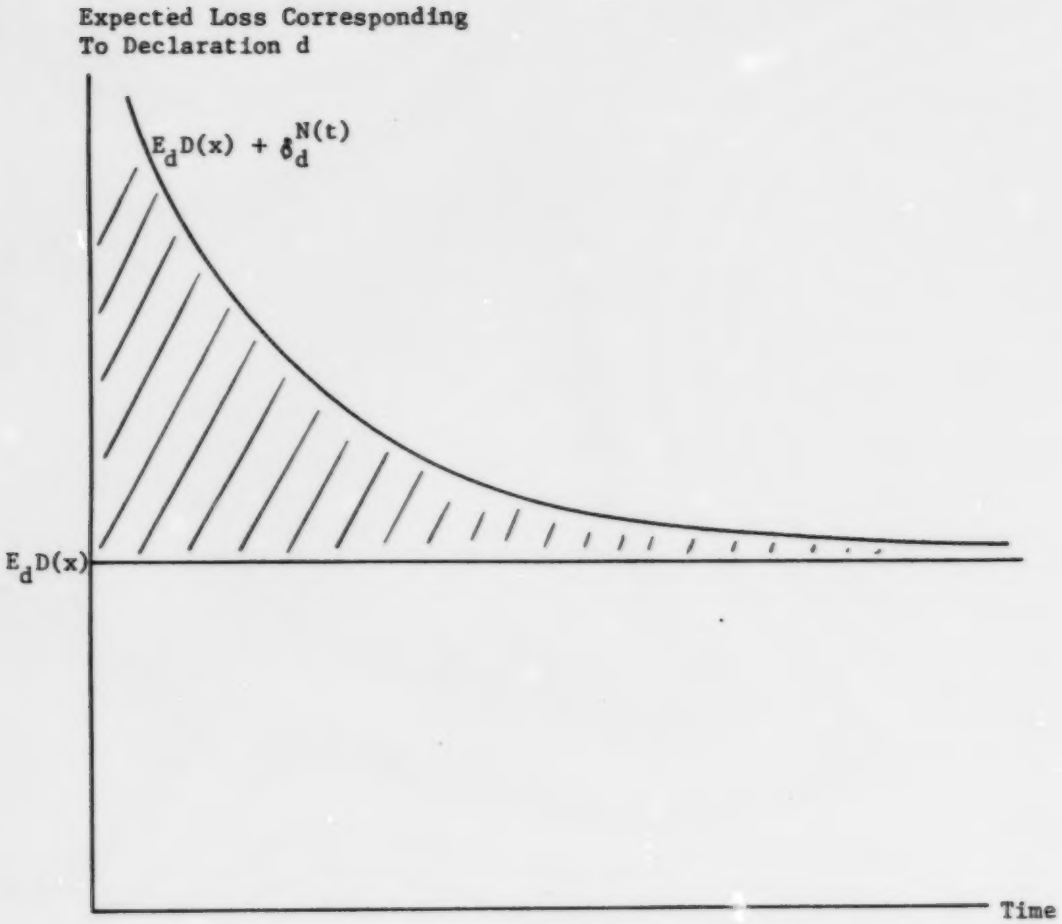


Figure 2 Graphical Representation of $E_d D(x) + \delta_d^{N(t)}$

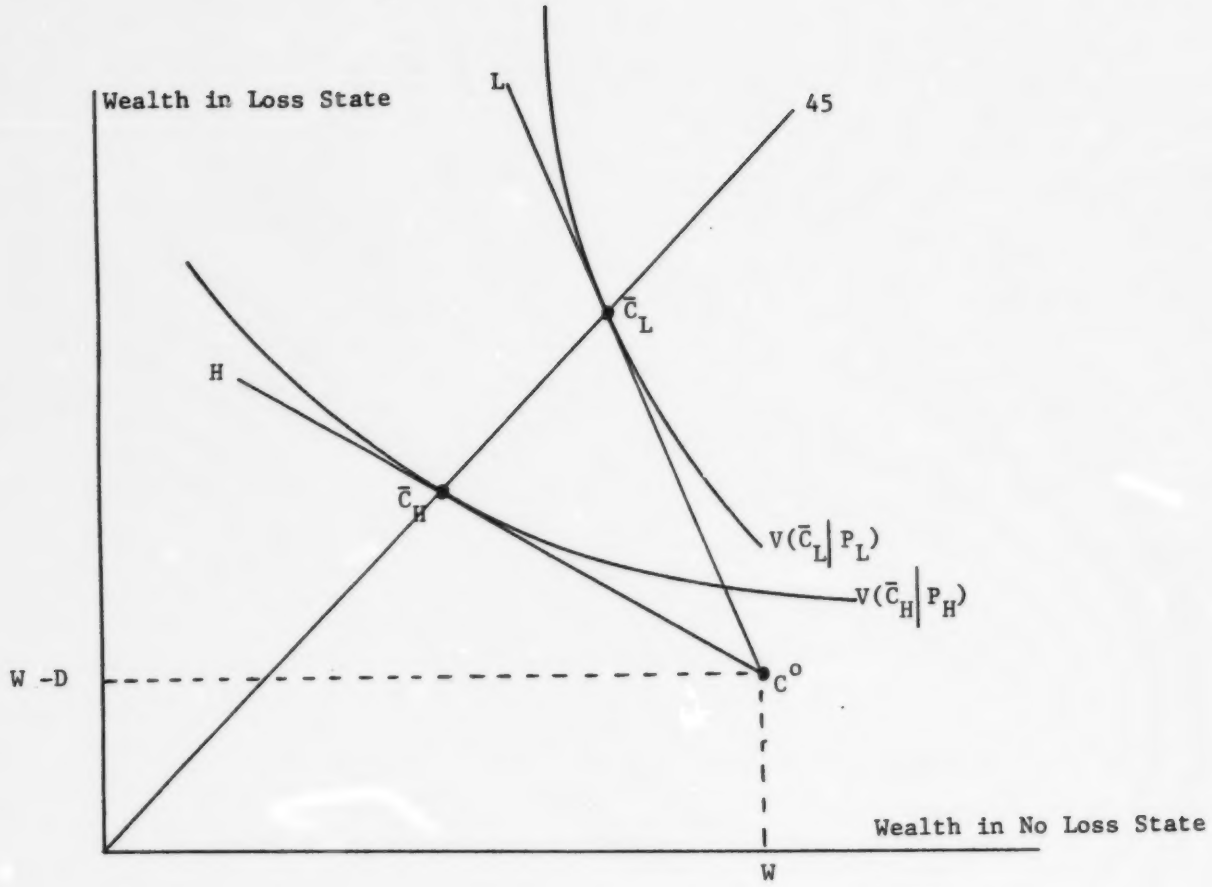
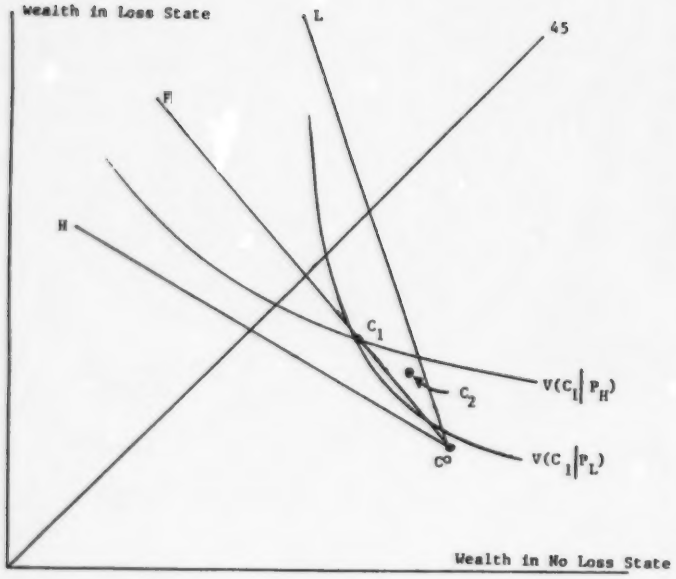
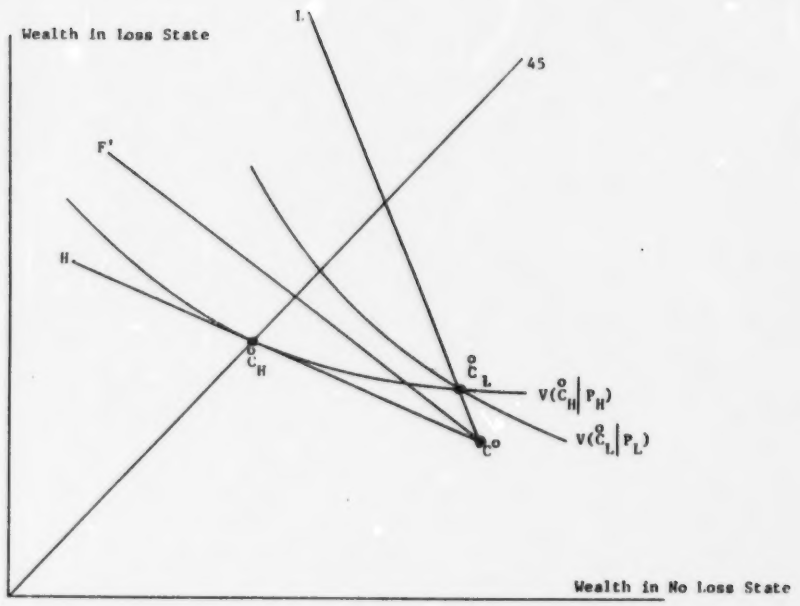


Figure 3 Full Information One-Period Contract



4a



4b

Figure 4 The Rothschild-Stiglitz Model

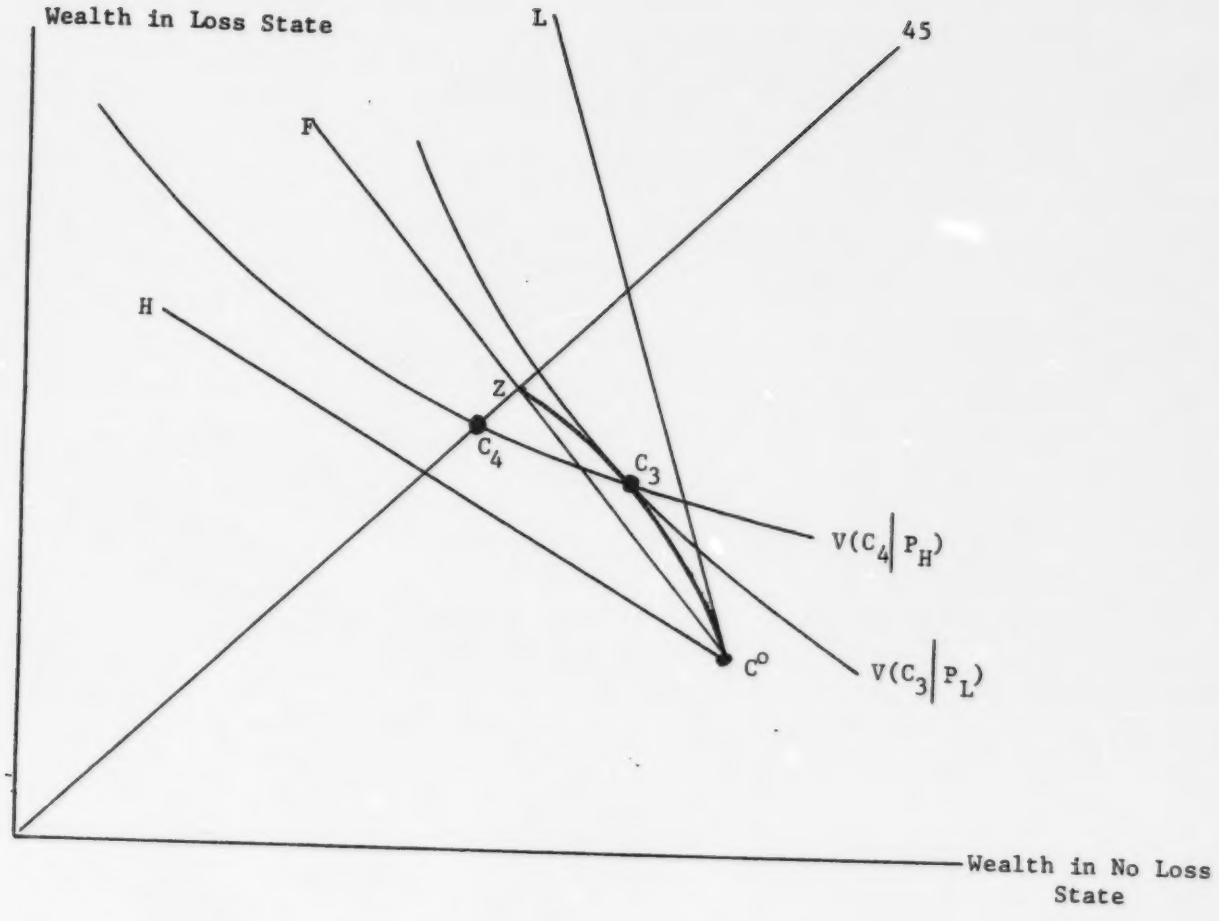


Figure 5 A Wilson-Miyazaki-Spence equilibrium with cross subsidization